MEASUREMENT OF DEBT SERVICING CAPACITY: AN APPLICATION OF DISCRIMINANT ANALYSIS*

Charles R. FRANK Jr. and William R. CLINE

Department of Economics, Princeton University

In the last ten years, there have been at least 21 instances of 11 countries which experienced such severe difficulties in servicing their international debt that they negotiated with creditor countries to postpone payments of interest or principal (see table 1). The amount of debt service rescheduled has been considerable, probably in the order of $2 billion. In some cases these negotiations were preceded by a period in which arrears of payments occurred (see Frank and Cline (1969) for more details).

The debt reschedulings have taken two different forms. The so-called Paris and Hague Club negotiations and the Ghanaian reschedulings are most often described as "ad hoc informal meetings of the major creditors." The creditor countries in these negotiations have done their best to maintain the idea that debt relief is not an institution but a very serious and unique event whenever it occurs. The recent trend toward the use of consortia, which are also responsible for the pledging and coordination of the regular flows of financial aid, has resulted in the erosion of the ad hoc concept. India, Indonesia, and Turkey are now in a situation which promises periodic rescheduling meetings.

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The difference in conception stems in part from differences in the structure of the debt. Many of the Latin American negotiations arose from difficulties encountered in servicing short- and medium-term commercial debt. The more recent phenomenon faced by consortia creditors is one in which long-term official lending forms a much more significant role in the debt-service burden. A rescheduling of payments over a one to five year period combined with some restrictions on the volume of commercial borrowing can significantly reduce the amount of debt service in the former case. In the long-term lending case, the debt must
be rescheduled over a considerable period of time to have any significant impact on the debt service burden.

It would be extremely useful for policy makers to have some indication of the difficulties which less developed countries are likely to have in the future. This would enable foreign aid donors to exercise caution in providing large amounts of assistance on hard terms to countries which are likely to have little future capacity to service foreign debt.

The purpose of this paper is to find an index or indicator of the likelihood that a less developed country will experience debt servicing difficulties. The indicator should satisfy two criteria: (1) it should be relatively simple, and (2) it should have a higher degree of predictability. The technique which we use is a modified form of discriminant analysis in which we take into account the differences in variability of the factors used to devise the index among debt rescheduling countries. It should be emphasized that the purpose of this paper is a rather modest one in which we try to determine the predictive performance of some widely used debt service indicators. Behind the composite index which we derive is an economic story which takes place in terms of stochastic money flows and accounting balances and surpluses.

1. The debt service ratio

The most commonly used indicator of debt servicing capacity is the debt service ratio. This ratio is defined as the ratio of service on debt to export earnings.\(^1\) The rationale for the use of the debt service ratio as an indicator of a country’s debt servicing capacity is that an increase in the debt service ratio indicates increased vulnerability to foreign exchange crises. Any shortfall in foreign exchange earnings or capital imports which is not covered by exchange reserves must be met by reducing imports: since debt service is a fixed obligation, the higher the debt service ratio, the greater is the relative burden on import reduction for a given shortfall in foreign exchange.

\(^1\) Modified versions of the debt service ratio include the ratio of debt service to earnings from exports of goods and services and the ratio of debt service to current account foreign exchange receipts. The ratio of debt service plus payments of income from equity investments to various definitions of exchange earnings is also used.
Unfortunately, the debt service ratio in and of itself is not a very good indicator of a country’s ability or lack of ability to pay its debts. The debt service ratio is merely an indicator of the proportion of foreign exchange earnings which are free to purchase imports. If exchange earnings are high relative to import demand, a high debt service ratio can be maintained. Furthermore, a country with good credit standing in international money markets may be able to finance a high debt service ratio, for a time at least, through a high level of borrowing.

The historical behavior of debt service ratios and instances of default also indicates an ability of some countries to tolerate high debt service ratios. Mexico and Israel have not defaulted nor requested debt rescheduling despite debt service ratios of 39 and 26 per cent respectively in recent years. Australia managed to avoid defaults on public and private debts with an investment service-exchange earnings ratio ranging from 43 to 44 per cent during the period 1930–1934. Canada avoided defaults and the imposition of exchange restrictions on current transactions with an investment service-exchange earnings ratio of 32 to 37 per cent over the 1931–1933 period. On the other hand, Bolivia, Brazil, Colombia, Cuba, Peru, and Uruguay defaulted in the period 1931–1933 with debt service ratios that were generally lower, on the order of 16 to 28 per cent (see Avramovic (1958), p. 194).

2. Other indicators

In order to devise a more appropriate index than the debt service ratio alone, we compiled data for 26 countries over the nine-year period 1960 to 1968 on eight factors (including the debt service ratio) which we felt might have an influence on the capacity to service debt (see Avramovic (1964)). Absence of data for specific years reduced the observations from a maximum of 234 (= 9·26) to 145 observations. The data included 13 reschedulings in 8 countries: Argentina, Brazil, Chile, Ghana, India, Indonesia, Turkey, and the U.A.R.

2 Using export figures from IMF and IBRD data on debt service for Mexico in 1967 and Israel in 1966.
3 See Mikesell (1968), p. 118. Investment service includes dividends on equity investments as well as debt service but the former is typically very small relative to debt service for most less developed countries today.
The first indicator of debt servicing difficulty is the ratio of debt service to exports, $X_1$. A one-year lag is specified in this and all subsequent $X$-variables, on the assumption that the debt service payment interruption in year $t$ occurs after decisions made near the end of year $t-1$ and that these decisions are based on the appearance of indicators during year $t-1$. Thus

$$X_{1t} = \frac{S_{t-1}}{\hat{E}_{t-1}}$$

(1)

where $S_{t-1}$ is debt service payments and $\hat{E}_{t-1}$ is “normal” exports in year $t-1$. “Normal” exports are used rather than actual exports in year $t-1$, under the assumption that authorities pay little attention to temporary highs or lows in exports but base decisions on what normal exports are expected to be. $\hat{E}_{t-1}$ was calculated as the “predicted” exports in year $t-1$, based on a regression of the logarithm of exports on time for the five-year period ending in year $t-1$. One should note that the debt service ratio was calculated on the basis of data on public debt and the publicly guaranteed private debt. Good data on private debt are not available.

The second indicator is $X_2$, the growth rate of exports. We assume that a country with a high export growth rate is less likely, ceteris paribus, to reschedule since the prospects are brighter for increasing foreign exchange earnings in the near future. The growth rate of exports is calculated on the basis of four-year averages over an eight-year period preceding the year of observation.

The variable $X_3$ is an export fluctuation index measured as the average absolute percentage deviation from an eight-year trend preceding the year of observation. We reasoned that a country with stable export earnings was less vulnerable to foreign exchange crises and could tolerate a higher debt service ratio.

The fourth variable $X_4$ is “non-compressible imports” as a fraction of total imports. It represents the degree to which imports may be reduced in time of balance-of-payments crisis. The higher this value, the more difficult it will be for a country to meet a debt servicing burden, and, therefore, the more likely debt rescheduling. Non-compressible imports were essentially intermediate goods, capital goods, and basic food-stuffs.

The fifth indicator $X_5$ is per capita income. It would seem likely that the lower per capita income, the less flexibility there would be for reducing consumption and thus, the more likely debt rescheduling.
The sixth indicator \( X_6 \) is the ratio of debt amortization to total outstanding debt (the inverse of the “average” maturity of loans). A low value for this indicator suggests that a country has predominantly long term debt liabilities. A country in this situation does not have very much shortrun flexibility in reducing in debt service commitments by temporary reduction of borrowing. Thus ceteris paribus such a country is more likely to reschedule. The absence of short-term liabilities also indicates that a country does not have significant access to short-term commercial credit facilities, i.e. the country is not particularly “credit-worthy”. A lack of a good credit reputation makes it difficult for a country to obtain quick access to additional credits when shortfalls in exchange earnings occur and rescheduling of debt becomes a more attractive alternative to alleviate foreign exchange crises.

The seventh variable \( X_7 \) is the ratio of imports to Gross National Product. A country with low imports relative to GNP is more likely to be able to withstand temporary import cuts than a country with high imports relative to GNP. Since \( X_4 \) already accounts for consumption import “compressibility”, \( X_7 \) may be thought of in the following terms. A country with high imports of intermediate inputs relative to GNP will find its production much more seriously threatened by inability to import than will a country which draws little of its intermediate inputs from imports. In sum, the higher \( X_7 \), the more likely a country is to require debt rescheduling.

Finally, the country’s reserves must be considered. Variable \( X_8 \) is the ratio of imports to reserves, where reserves include gold reserves, holding of dollars or sterling, and net position at the IMF. \(^4\) Other influences equal, the country with high reserves relative to imports is unlikely to be in need of debt rescheduling.

The data sources for the indicators were as follows. Debt service amortization and outstanding debt were taken from studies by the IBRD, Avramovic (1964), the Development Assistance Committee of the OECD, and AID country files. An attempt was made to find ex ante debt service for the year in which rescheduling occurred. In cases where these data were available, they were used in place of debt service in the

\(^4\) That is, the ceiling of permissible borrowings less the amount of borrowings already incurred.
prior year: in variable $X_1$, "expected" service in year $t$ replaced observed service in year $t-1$.  

Population, dollar values of exports, imports, and reserves (gold and foreign exchange plus net IMF position) were taken from standard international statistical sources. Gross national product in current U.S. dollars was derived from AID country files. The "non-compressible" imports were calculated from the SITC breakdowns of imports and include cereals (SITC group 4) and raw materials and manufactured goods in SITC groups 2 through 7, excluding passenger cars, textile yarn, and finished paper goods. Thus, the consumer goods considered "compressible" were foods other than cereals, beverages and tobacco (SITC group 1), the exceptions to groups 2 through 7 mentioned above, and other manufactured goods (SITC group 8), such as clothing.

3. Discriminant analysis

Let $Z$ stand for the composite index of debt servicing capacity. We denote by $\pi_1$ the population of rescheduling countries (countries which have reached the limit of their debt servicing capacity) and by $\pi_2$, the population of countries which have not reached the limit of their debt servicing capacity. Let $x$ stand for the vector of observations of $n$ indicators of debt servicing capacity that are to be combined into the composite index. That is, $Z = f(x)$. We wish to find the function $Z = f(x)$ and a critical value $Z^*$ of this function (or index) such that if $Z = f(x) \geq Z^*$, we classify a country as coming from population $\pi_1$ and if $Z = f(x) < Z^*$, we classify a country as coming from $\pi_2$. Let $q_1$ be the a priori probability that a country comes from $\pi_1$ and $q_2$ the a priori probability that a country comes from $\pi_2$.

For any function $f(x)$ and critical value $Z^*$, there are two kinds of errors which can be made. A type I error results when a country actually

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5 That is, the use of debt service in year $t-1$ to calculate $X_{1t}$ was made on the assumption that this value approximated the expected level of debt service in year $t$ which would occur in the absence of rescheduling. Thus, in rescheduling cases in which the actual anticipated debt service for year $t$ (if there were to be no rescheduling) was known, it was used in the calculation $X_{1t}$.

6 See United Nations Yearbook. For several countries, complete time series from 1957 to 1967 were not available. In these cases data for the missing years were estimated by substituting the non-compressible fraction of total imports for the closest three-year average for which data were available.
belonging to \( \pi_1 \) is classed in \( \pi_2 \), and a type II error results when a country actually belonging to \( \pi_2 \) is classed in \( \pi_1 \). The expected cost of making errors is

\[
C = q_1 C(I) p(I) + q_2 C(II) p(II)
\]  

(2)

where \( C(I) \) and \( C(II) \) are the costs of making type I and type II errors, respectively and \( p(I) \) and \( p(II) \) are the probabilities of making type I and type II errors, respectively. We wish to choose a function \( f(x) \) and a critical value \( Z^* \) such that the expected cost of errors is minimized (a Bayesian procedure). Alternatively, if \( q_1 \) and \( q_2 \) are not known we choose an \( f(x) \) and \( Z^* \) so as to minimize the maximum of \( C(I) p(I) \) and \( C(II) p(II) \), i.e., a minimax solution.

3.1. The Bayesian approach

If we know \( q_1 \) and \( q_2 \) and if the vector \( x \) is normally distributed, it is possible to show (see Anderson and Bahadur (1962)) that \( f(x) \) which minimizes the expected cost of errors is the quadratic form

\[
Z = f(x) = x' A x + B x + C
\]  

(3)

where \( A \) is an \( n \times n \) matrix, \( B \) is a \( n \)-dimensional column vector and \( C \) is a constant.

\[
A = (\Sigma_2^{-1} - \Sigma_1^{-1}) / 2
\]  

(4)

\[
B = \mu_1' \Sigma_1^{-1} - \mu_2' \Sigma_2^{-1}
\]  

(5)

\[
C = \frac{1}{2} \mu_2' \Sigma_2^{-1} \mu_2 - \frac{1}{2} \mu_1' \Sigma_1^{-1} \mu_1 + (\log | \Sigma_2 | - \log | \Sigma_1 |) / 2
\]  

(6)

\( \Sigma_2 \) and \( \Sigma_1 \) are the covariance matrices of \( x \) in \( \pi_2 \) and \( \pi_1 \), respectively; and \( \mu_2 \) and \( \mu_1 \) are the means of \( x \) in \( \pi_2 \) and \( \pi_1 \), respectively. It is easy to see that if \( \Sigma_2 = \Sigma_1 \), \( f(x) \) becomes a linear function. The critical value of \( Z \) is

\[
Z^* = \log_e [ q_2 C(II)/q_1 C(I) ]
\]  

(7)

Although, one generally does not know the population means \( \mu_1 \) and
\( \mu_2 \) and covariance matrices \( \Sigma_1 \) and \( \Sigma_2 \), \( f(x) \) can be estimated by estimating the means and covariances from a sample of observations from \( \pi_1 \) and \( \pi_2 \).

3.2. The minimax approach

If we do not know \( q_1 \) and \( q_2 \), if the vector \( x \) is normally distributed in \( \pi_1 \) and \( \pi_2 \), and if we restrict ourselves to linear functions \( f(x) = \beta x \) where \( \beta \) is an \( n \)-dimensional vector, the minimax solution is obtained (see again, Anderson and Bahadur (1962)) from

\[
\max_{\lambda, \beta} \left[ \frac{\lambda \beta (\mu_1 - \mu_2)}{(\beta \Sigma_2 \beta')^{1/2}} \right] \tag{8}
\]

subject to

\[
\lambda = \frac{(\beta \Sigma_2 \beta')^{1/2}}{(\beta \Sigma_2 \beta')^{1/2} + (\beta \Sigma_1 \beta')^{1/2}}. \tag{9}
\]

Note that \( (\beta \Sigma_2 \beta') \) is the variance of the function \( Z \) in \( \pi_2 \) and \( (\beta \Sigma_1 \beta') \) is the variance of \( Z \) in \( \pi_1 \).

The value of \( Z^* \) is given by

\[
Z^* = \lambda \beta \mu_1 + (1 - \lambda) \beta \mu_2. \tag{10}
\]

It is possible to show by differentiating (7) with respect to \( \beta \) that the solution to (8) and (9) are the \( \beta \) and \( \lambda \) which solve

\[
[\lambda \Sigma_1 + (1 - \lambda) \Sigma_2] \beta' = (\mu_1 - \mu_2) \tag{11}
\]

and (9).

There are several different approximative methods which one might use to solve eqs. (11) and (9). Anderson and Bahadur (1962) propose the following techniques. Note from (9) that \( 0 \leq \lambda \leq 1 \). One can simply divide the interval \([0, 1]\) into an equal number of subintervals and choose the midpoint \( \lambda_i \) of the \( i \)th interval, solve (11) for \( \beta \), and determine the difference between \( \lambda_i \) and \( \lambda \) as given by (9). The approximate solution is that \( \lambda_i \) which results in the smallest absolute difference between \( \lambda_i \) and \( \lambda \) as given by (9).
Another method (the one which we use) is to choose an initial $\beta$, solve (9) for $\lambda$, substitute this value of $\lambda$ into (11), solve (11) for a new $\beta$, and repeat the process. This iterative process converged fairly rapidly on all the samples which we used.

4. Results

4.1. Linear discriminant functions – equal covariance

The first step of the analysis involved a determination of the linear discriminant function using all of the indicators. In this first pass, we assumed equal covariance matrices. The resulting discriminant function was

$$Z = -0.219 + 1.4955 X_1 - 0.0545 X_2 - 0.0724 X_3$$

$$- 0.1121 X_4 + 0.0152 X_5 - 1.3739 X_6$$

$$+ 0.2655 X_7 + 0.0360 X_8$$

(5.16) \hspace{1cm} \begin{array}{ccc}
(-0.13) & (-0.11) \\
(0.59) & (1.01) & (-3.27) \\
(0.70) & (3.60) 
\end{array}

The critical value of $Z$ is $Z^* = 0$. 7 The number of type I errors was 3 and the number of type II errors was 15. The percentage of errors was 12.4 per cent of a sample of 145 country years.

Although the assumptions of regression analysis are not appropriate in discriminant analysis, we found it useful to apply the usual linear regression tests to obtain some notion of the relative importance of the various variables. (The $t$-ratios are in parentheses in eq. (12).) The most striking result was the dominance of only three variables: the debt service ratio ($X_1$), the amortization/debt ratio ($X_6$) and the imports/reserve ratio ($X_8$). Only these three variables were statistically significant at the 5 per cent level.

7 Assuming a priori probabilities $q_1$ and $q_2$ are equal and the costs $C(I)$ and $C(II)$ of misclassification are equal, Eq. (12) was estimated by a simple linear regression $Y_t = \beta X + \beta_0$ where $Y_t = 1$ if the $t$th observation comes from population $\pi_1$ and $Y_t = 0$ if from population $\pi_2$. 
The results of the discriminant function (12) indicated that one could do fairly well using only three significant variables. Furthermore, it was clear that the covariance matrices estimated in the two populations were quite different. Therefore, all further experiments on the discriminant functions were confined to three variables, and quadratic functions (based on unequal covariances) as well as iterated linear functions (also based on unequal covariances) were estimated.

The iterated linear discriminant functions were run both with and without the variable \( X_8 \) since we wished to use the discriminant function for predictive purposes, and prediction of the reserves to imports ratio is particularly difficult. The iterated linear functions seemed to converge after about 10 iterations. The results were as follows:

<table>
<thead>
<tr>
<th>Linear coefficients of variables</th>
<th>Critical value</th>
<th>Iteration</th>
</tr>
</thead>
<tbody>
<tr>
<td>( X_1 ) ( X_6 ) ( X_8 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>26.779 ( -24.824 ) 0.702</td>
<td>6.412</td>
<td>1</td>
</tr>
<tr>
<td>19.750 ( -1.107 ) 0.390</td>
<td>5.295</td>
<td>10</td>
</tr>
<tr>
<td>27.285 ( -20.380 ) –</td>
<td>3.437</td>
<td>1</td>
</tr>
<tr>
<td>19.029 ( -0.024 ) –</td>
<td>3.423</td>
<td>10</td>
</tr>
</tbody>
</table>

The first iteration of the two- and three-variable discriminant functions are linear discriminant functions computed on the basis of an assumption of equal covariances in the two populations. Note that the relative influence of the \( X_6 \) variable (the ratio of amortization to debt or the inverse of the "average" maturity of debt outstanding) declines as the assumption of unequal covariance structure is incorporated in the linear function estimated at iteration number 10. This occurs because the estimated variance of the amortization/debt ratio is quite high among rescheduling countries and less weight is therefore placed on this variable when differences in variance are taken into account.

The quadratic discriminant function was also estimated both with and without the variable \( X_8 \). The coefficients of the quadratic quadratic form shown in eq. (3) were as follows for the two- and three-variable cases:

Three-variable case: \( X_1, X_6, X_8 \)

\[
A = \begin{bmatrix}
31.008 & 179.424 & 0.625 \\
-179.424 & -65.214 & -1.910 \\
-0.625 & -1.910 & 0.127
\end{bmatrix}
\]

\[
B = (52.850, 97.612, -0.041)
\]

\[
C = -13.695
\]
Two-variable case: $X_1, X_6$

$$A = \begin{bmatrix} 35.606 & -171.396 \\ -171.396 & -54.369 \end{bmatrix}$$

$$B = (42.122, 73.070)$$

$$C = -9.341$$

If we assume equal costs of mis-classification and equal a priori probabilities, the critical value of the quadratic discriminant function is zero. In order to determine, however, the effects of changes in the critical value, parametric variations were run to determine which critical value gave the fewest type I and type II errors. The best critical value from this point of view was $-0.600$ in the three variable case and $+0.302$ in the two variable case. (This procedure is not legitimate in the case of the iterative linear discriminant functions since the form of the discriminant function varies as the critical value of the function varies.)

Comparison of the type I and type II errors for the iterative linear functions and the quadratic functions is given in the following table (percentage errors within groups are given in parentheses):

<table>
<thead>
<tr>
<th>Case</th>
<th>Type I</th>
<th>Type II</th>
</tr>
</thead>
<tbody>
<tr>
<td>Three-variable case — Linear</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Iteration 1</td>
<td>3 (23.1)</td>
<td>14 (10.6)</td>
</tr>
<tr>
<td>Iteration 10</td>
<td>1 (7.7)</td>
<td>25 (19.9)</td>
</tr>
<tr>
<td>Two-variable case — Linear</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Iteration 1</td>
<td>1 (7.7)</td>
<td>17 (12.8)</td>
</tr>
<tr>
<td>Iteration 10</td>
<td>0 (0)</td>
<td>26 (19.7)</td>
</tr>
<tr>
<td>Three-variable case — Quadratic</td>
<td>1 (7.7)</td>
<td>21 (15.8)</td>
</tr>
<tr>
<td>Two-variable case — Quadratic</td>
<td>0 (0)</td>
<td>12 (9.0)</td>
</tr>
</tbody>
</table>

Type I errors occur when a rescheduling country is predicted as non-rescheduling and type II errors occur when a non-rescheduling country is predicted as rescheduling. There were a total of 13 observations of rescheduling countries and 132 on non-rescheduling countries.

Of all the linear discriminant functions, the best in terms of the average number of errors in the two groups is the first iteration of the two-variable, linear function. Of the two quadratic discriminant functions, the best is the two-variable, quadratic function. Furthermore 13 of the 17 type II errors from the best linear function are for observations from countries which in nearby years did in fact have reschedu-
ulings. Argentina was predicted to reschedule in 1962 and 1964 when actually reschedulings took place in 1961, 1963, 1965. Brazil, Chile, India and Turkey are also predicted to reschedule in years immediately preceding reschedulings. The other 4 type II errors relate to Mexico which has had a very high debt service ratio but has not rescheduled. The two-variable quadratic discriminant function, however, makes only one type II error for Mexico (in 1962). One error relates to Colombia (1964) when the debt service ratio was around 20 per cent and the remaining 10 type II errors are errors in timing.

The critical values of the discriminant function may be converted to a critical debt service ratio once the average maturity of debt is known. For example, if the average maturity of debt for a particular country is 30 years (typical of a country like India), the critical debt service ratio is 15 per cent from the first iteration of the two-variable linear function. If, however, the average maturity of debt is 5 years (typical of a country like Mexico), the critical debt service ratio is 27.8 per cent. In general, the longer the average maturity of debt, the lower is the critical debt service ratio. This is illustrated in fig. 1 in which the straight line AB shows the various combination of debt service ratios and average maturity of debt (shown in parentheses along the horizontal axis) which result in a composite index equal exactly to the critical value for the first iteration of the two-variable linear discriminant function. The points $P_1$ and $P_2$ refer to the representative points mentioned above (30 and 5 years average maturity respectively). If the combination of debt-service ratio and average maturity of debt lie above the line AB, a country is likely to reschedule, if below the line AB, a country is more likely not to reschedule.

The curve CD shows the relevant part of the two-variable quadratic discriminant function for a critical value of 0.3. If the combination of the debt-service ratio and average maturity of debt lie above the hyperbola CD, the likelihood is great of a rescheduling. The hyperbola indicates that any country with a debt service ratio above 19 per cent is very likely to reschedule except when the average maturity of debt is very low.

Some actual observations are shown in fig. 1. $T_{1965}$ and $C_{1966}$ refer to Turkey in 1965 and Ghana in 1966, two country-years in which reschedulings actually took place. $P_{1966}$ and $N_{1967}$ are Pakistan in 1966 and Nigeria in 1967, two country-years in which no reschedulings took place. $M_{1965}$ is Mexico in 1965, an interesting observation. No rescheduling took place but with the linear discriminant function, Mexico
is predicted as rescheduling in 1965, a type II error. With the quadratic discriminant function, however, Mexico in 1965 is correctly predicted as non-rescheduling.

5. Projections of debt servicing capacity

In order to project the discriminant functions, we made projections of total debt, interest payments, amortization payments, total debt service, and debt service ratios for 17 countries from 1967 to 1992. The 17 countries are:

<table>
<thead>
<tr>
<th>Argentina</th>
<th>Peru</th>
<th>Bolivia</th>
<th>Dominican Republic</th>
<th>Mexico</th>
</tr>
</thead>
<tbody>
<tr>
<td>Turkey</td>
<td>Korea</td>
<td>India</td>
<td>Pakistan</td>
<td>Indonesia</td>
</tr>
<tr>
<td>Chile</td>
<td>Iran</td>
<td>Nigeria</td>
<td>Pakistan</td>
<td></td>
</tr>
<tr>
<td>Colombia</td>
<td>Nigeria</td>
<td>Tunisia</td>
<td>Brazil</td>
<td></td>
</tr>
</tbody>
</table>

Fifteen of these countries rank in the top 20 in terms of total foreign
assistance received in the last decade. Many of them have already experienced debt servicing difficulties. Note that Ghana and the United Arab Republic, although heavily involved in payment of debt service, are not included because of lack of data.

The foundation of the projections was, in most cases, IBRD estimates of service payments due during the period 1967–1992 on the basis of debt already outstanding at the beginning of 1967. Upon this foundation, we assumed new loan disbursements to these countries to continue at the same gross amount (or as an alternative, at the same net amount) as it had in the recent past (in most cases the last two or three years). The new lending was broken down into several terms categories usually from 4 to 7 categories) based on recent experience in borrowing by source. The most recent set of loan terms of each lending source, e.g., A.I.D. development loans or IBRD loans, were then applied to the appropriate categories.

The debt service ratio projections were based on three alternative assumptions about export growth: (i) a continuation of the 1960–1967 export growth trend for each individual country, (ii) a four per cent rate of export growth, and (iii) an eight per cent rate of export growth.

Projections were run for a number of the two- and three-variable discriminant functions. Table 2 gives percentages of country years for three groups of countries using the two-variable quadratic discriminant function. Group I countries are likely to be faced with serious debt servicing problems regardless of the aid assumption used or the export growth rate. Group II countries would face serious problems only if export growth is low and a large volume of capital imports continue. Group III countries should not face serious problems under the various assumptions made if the quadratic function is a good index of debt servicing difficulty.

For purposes of comparison the numbers in parentheses in table 2 give percentages of country-years using the first iteration of the two-variable linear discriminant function. This function indicates a greater percentage of difficulty for all countries under the assumptions made. The big difference between the linear and quadratic functions occurs

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8 The IBRD data includes all debt which is payable to creditors outside the debtor country with an original maturity of one year or more which are obligations of governments or public agencies. However, it includes publicly guaranteed private debt. It does not include: (1) transactions with the International Monetary Fund, (2) non-guaranteed private debt, (3) local currency obligations, and (4) other minor categories of debt.
Table 2
Projections of discriminant functions:
percentage of country-years for which serious debt servicing problem is indicated
1968 - 1992

<table>
<thead>
<tr>
<th>Country group</th>
<th>Constant gross aid export growth rate</th>
<th>Constant net aid export growth rate</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1960-1967 trend</td>
<td></td>
</tr>
<tr>
<td></td>
<td>4 per cent</td>
<td>8 per cent</td>
</tr>
<tr>
<td></td>
<td>4 per cent</td>
<td>8 per cent</td>
</tr>
<tr>
<td>I</td>
<td></td>
<td></td>
</tr>
<tr>
<td>India</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Indonesia</td>
<td>87</td>
<td>99</td>
</tr>
<tr>
<td>Pakistan</td>
<td>48</td>
<td>79</td>
</tr>
<tr>
<td>Tunisia</td>
<td>(100)</td>
<td>(100)</td>
</tr>
<tr>
<td></td>
<td>(79)</td>
<td>(100)</td>
</tr>
<tr>
<td></td>
<td>(100)</td>
<td>(100)</td>
</tr>
<tr>
<td>II</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Brazil</td>
<td>1</td>
<td>16</td>
</tr>
<tr>
<td>Chile</td>
<td>2</td>
<td>45</td>
</tr>
<tr>
<td>Columbia</td>
<td>16</td>
<td>1</td>
</tr>
<tr>
<td>Dom, Rep.</td>
<td>2</td>
<td>(31)</td>
</tr>
<tr>
<td>Israel</td>
<td>16</td>
<td>(47)</td>
</tr>
<tr>
<td>Korea</td>
<td>16</td>
<td>(13)</td>
</tr>
<tr>
<td>Peru</td>
<td>16</td>
<td>(37)</td>
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<tr>
<td>Turkey</td>
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<td>(88)</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>(32)</td>
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<tr>
<td>III</td>
<td></td>
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<tr>
<td>Mexico</td>
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<td>0</td>
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<tr>
<td>Argentina</td>
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<td>0</td>
</tr>
<tr>
<td>Bolivia</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Iran</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Nigeria</td>
<td>(46)</td>
<td>(21)</td>
</tr>
<tr>
<td></td>
<td>(11)</td>
<td>(14)</td>
</tr>
<tr>
<td></td>
<td>(51)</td>
<td>(26)</td>
</tr>
</tbody>
</table>

Figures in parentheses refer to first iteration of two-variable linear discriminant function. Other figures refer to two-variable quadratic function.

for Argentina, Mexico and Israel. These countries tend to have high debt-service ratios but a short term-structure of their debt. The quadratic function takes the term structure of debt more into account and tends to project much less debt servicing difficulty for these countries.

7. Conclusions

The experiments with discriminant analysis show that it is possible to obtain a very high prediction rate using only two factors, the debt service ratio and the average maturity of debt. The best predictions are obtained when it is assumed that covariance matrices are unequal.
Projections of the debt service indicator for 17 major countries show that over the next decade the likelihood is high that many of them will seek debt relief and for four of these countries, the difficulties in servicing debt will be exceptionally serious. These projections assume, however, that roughly the same factors which operated in the past are likely to continue in the future.

There may, however, be mitigating circumstances in the future that will make the debt service burden less onerous. First, an increasing number of foreign aid recipients are serviced by consortia of aid donors who are likely to take into account the debt service burden when setting aid levels and policies. Secondly, the build up of debt service is relatively easy to foresee given the availability of data on government and government guaranteed loans outstanding. Appropriate policies for adjusting to the increased level of debt service can be initiated well ahead of time. In many past rescheduling exercises, debt servicing difficulties arose suddenly and without warning as the result of excessive reliance on short-term, non-guaranteed exports credits for which little data were available.

On the other side, however, there are many reasons to believe that debt servicing difficulties will be even more severe than our projections indicate. First, both the constant gross aid and constant net aid assumptions used in the debt service projections may be conservative in the light of past experience and in view of reasonable estimates of LDC need for foreign capital. For example, between 1960–62 and 1965–67 gross aid from members of the Development Assistance Committee to the LDC’s and multilateral agencies grew at 4.2 per cent annually and aid net of amortization and interest grew at 2.5 per cent.

Secondly, the debt service projections assume that terms of foreign lending will be roughly the same in the future as in the 1964–66 period. Several recent developments will very likely invalidate that assumption and cause a greater burden of debt service. United States terms on development loans have hardened substantially since 1964. 9 PL 480 assistance is gradually being shifted to a hard currency repayable basis by 1971 with terms similar to development loans. Another development is a prospective rapid increase in lending by countries (e.g. Germany and

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9 In 1964, these terms were 1.0 per cent interest during a ten year grace period for repayment of principal and 2.5 per cent over the remaining life of the loan. In 1968 these two rates were 2.0 and 3.0 per cent, respectively.
Japan), multilateral agencies (e.g. the World Bank, and the Inter-American Development Bank) and other institutions (e.g. the U.S. Export-Import Bank) which lend on near-commercial terms. Revised and more recent data which were just becoming available at the time this paper was written indicate that for a number of countries in groups I and II, the debt service projections for 1967 and 1968 are underestimates because of the hardening of average terms.

Thirdly, the data and projections for debt service exclude non-guaranteed export credits which can add significantly to the debt service burden and cause "lumpiness" in debt service payments.

On balance, then, the prospects for serious debt servicing difficulties implicit in the analysis probably err on the optimistic rather than the alarmist side. Furthermore, it is significant to note that many of the countries in groups I and II have a debt structure including many long term, low interest loans, notwithstanding their high level of debt service. It will be difficult for these countries to work themselves out of a high debt service situation by a temporary restriction on commercial borrowing.

References

United Nations, Yearbook of International Trade Statistics, various years.